

Nonregenerative MIMO Relaying with Optimal Transmit Antenna Selection

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Abstract

We derive optimal SNR-based transmit antenna selection rules at the source and relay for the nonregenerative half duplex MIMO relay channel. While antenna selection is a suboptimal form of beamforming, it has the advantage that the optimization is tractable and can be implemented with only a few bits of feedback from the destination to the source and relay. We compare the bit error rate of optimal antenna selection at both the source and relay to other proposed beamforming techniques and propose methods for performing the necessary limited feedback.

I. INTRODUCTION

Despite the lack of precise knowledge of its basic theoretical behavior and limits, relaying is beginning to find practical application in standards such as IEEE 802.16j [1]. By deploying relatively inexpensive relays, service providers can reduce the number of base stations required to serve a given area, or increase capacity at the cell edge.

Relaying research efforts have also increased recently [2]–[7]. Capacity bounds for the full-duplex MIMO relay channel were derived in [2], [3]. The authors of [6] derive the optimal infinite-SNR diversity-multiplexing tradeoff for the half duplex MIMO relay channel and find that a compress-and-forward strategy is optimal in this sense. Recently, practical strategies have been developed for MIMO relaying. Both [4] and [7] derive the mutual-information-maximizing nonregenerative linear relay for spatial multiplexing when the direct link is ignored.

This letter derives the optimal transmit antenna selection criteria at both source and relay; i.e., all transmissions occur using the transmit antenna that will give the destination the highest post-processing signal-to-noise ratio. We consider the case where only a single spatial stream is to be sent from source

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to destination. This scenario arises when the channel is ill-conditioned (i.e., there is a dominant path of propagation in the source-destination channel), or if robustness via diversity is preferred over throughput (i.e., near the cell edge).

Unlike most previous practical MIMO relay results (e.g., [4], [5], [7]), the strategy derived here is the optimal transmit antenna selection strategy when the direct link from source to relay is not ignored. We prove that transmit antenna selection, combined with an MMSE receiver at the destination, achieves the full diversity order of the MIMO single relay channel. That is, at high SNR the probability of outage decays with SNR as quickly as is possible in such a model. Further, antenna selection requires less feedback than beamforming. Distributed space-time codes, which may also achieve the full diversity gain, not only require their own level of overhead for coordination and synchronization, but also require the relay to be able to decode the message transmitted by the source.

Compared to recent results using limited feedback beamforming [8], under the tested parameters given in the aforementioned paper, antenna selection at both source and destination is about twice as likely to cause bit errors as a Grassmannian codebook with 16 codes, which is a loss of about 1 dB at high SNR. In return, antenna selection requires only $\log_2 N_S N_R$ bits of feedback versus $3 \log_2 N + 2b$ bits in [8], where N_S and N_R are the number of antennas at the source and relay, respectively, N is the size of the Grassmannian codebook, and b is the quantization in bits of the SNR feedback required in [8].

This letter uses capital boldface letters to refer to matrices and lowercase boldface letters for column vectors. The notation $\|\mathbf{h}\|$ refers to the L2-norm of the vector \mathbf{h} , and \mathbf{H}^* is the complex conjugate transpose of the matrix \mathbf{H} . The vector $\mathbf{h}^{(i)}$ refers to the i th column of the matrix \mathbf{H} . Finally,

$$A \doteq B \iff \lim_{\text{SNR} \rightarrow \infty} \frac{\log A}{\log \text{SNR}} = -B.$$

II. SYSTEM MODEL & ANTENNA SELECTION

We assume a single source \mathcal{S} transmitting information to a destination \mathcal{D} with a single relay \mathcal{R} aiding the transmission. The source, destination, and relay are equipped with N_S , N_D , and N_R antennas, respectively. All nodes operate in half-duplex mode. Unlike most prior work in MIMO relaying, *we do not ignore the direct link between \mathcal{S} and \mathcal{D} .*

The source \mathcal{S} wishes to transmit the scalar symbol s to \mathcal{D} , where $\mathbb{E}|s|^2 = E_s = \text{SNR}$, E_s is the average power constraint at both \mathcal{S} and \mathcal{R} , and $\sigma^2 = 1$ is the overall noise power at each node. Since the signal-to-noise ratio is the metric of interest, an imbalance of noise energy among the nodes can be modeled in the appropriate fading parameter for \mathbf{H}_{XY} . For instance, if the relay has noise power

σ_r^2 , in an independent Rayleigh fading environment these definitions would change the channel fading parameter of the corresponding exponential distribution from λ_{SR} to $\lambda_{SR}\sigma_r^2$.

We denote the channel from \mathcal{X} to \mathcal{Y} , $\mathcal{X} \in \{\mathcal{S}, \mathcal{R}\}$, $\mathcal{Y} \in \{\mathcal{R}, \mathcal{D}\}$, $\mathcal{X} \neq \mathcal{Y}$, as \mathbf{H}_{XY} , and $\mathbf{h}_{XY}^{(i)}$ is the vector channel from the i th transmit antenna at \mathcal{X} to \mathcal{Y} . We also define

$$\gamma_{XY}^{(i)} = \left\| \mathbf{h}_{XY}^{(i)} \right\|^2 \text{SNR} \quad (1)$$

to be the equivalent receive SNR from $\mathcal{X}_i \rightarrow \mathcal{Y}$.

We assume the block fading model. In the first stage, if \mathcal{S} transmits s on antenna i , \mathcal{R} receives the signal

$$\mathbf{y}_R = \mathbf{h}_{SR}^{(i)} s + \mathbf{n}_R, \quad (2)$$

where \mathbf{n}_R is the zero-mean spatially white complex Gaussian noise vector with covariance $\sigma^2 \mathbf{I}_{N_R}$ as observed by \mathcal{R} . Since the relay is also transmitting on only one of its antennas, it must combine its received vector to form a single symbol. It can be shown that the optimal way to do this is to perform MRC on the signal, resulting in a scalar

$$s_R = \alpha (\mathbf{h}_{SR}^{(i)})^* \mathbf{y}_R, \quad (3)$$

where α is the scaling factor to ensure \mathcal{R} transmits at its expected power constraint; i.e.,

$$\alpha^2 = \frac{1}{\left\| \mathbf{h}_{SR}^{(i)} \right\|^4 + \left\| \mathbf{h}_{SR}^{(i)} \right\|^2 / \text{SNR}}. \quad (4)$$

At \mathcal{D} , the first stage results in

$$\mathbf{y}_{D,1} = \mathbf{h}_{SD}^{(i)} s + \mathbf{n}_{D,1}. \quad (5)$$

In the second stage, \mathcal{R} transmits s_R to \mathcal{D} on antenna k :

$$\mathbf{y}_{D,2} = \mathbf{h}_{RD}^{(k)} s_R + \mathbf{n}_{D,2}. \quad (6)$$

The destination now has two observations containing s . To put the channel in standard MIMO notation, we define

$$\mathbf{h} = \begin{pmatrix} \mathbf{h}_{SD}^{(i)} \\ \frac{\left\| \mathbf{h}_{SR}^{(i)} \right\| \mathbf{h}_{RD}^{(k)}}{\sqrt{\left\| \mathbf{h}_{SR}^{(i)} \right\|^2 + 1 / \text{SNR}}} \end{pmatrix} \quad (7)$$

$$\mathbf{n} = \begin{pmatrix} \mathbf{n}_{D,1} \\ \frac{\mathbf{h}_{RD}^{(k)} (\mathbf{h}_{SR}^{(i)})^* \mathbf{n}_R}{\left\| \mathbf{h}_{SR}^{(i)} \right\| \sqrt{\left\| \mathbf{h}_{SR}^{(i)} \right\|^2 + 1 / \text{SNR}}} + \mathbf{n}_{D,2} \end{pmatrix} \quad (8)$$

$$\mathbf{y}_D = \begin{pmatrix} \mathbf{y}_{D,1} \\ \mathbf{y}_{D,2} \end{pmatrix} \quad (9)$$

$$\gamma^{(i)} = \gamma_{SD}^{(i)} \left(\frac{\gamma_{SD}^{(i)}(\gamma_{SR}^{(i)} + 1)^2 + \gamma_{SR}^{(i)}\gamma_{RD}^{(k)}(\gamma_{SR}^{(i)} + 1 + \gamma_{SR}^{(i)} + 1 + \gamma_{SR}^{(i)}\gamma_{RD}^{(k)})}{\gamma_{SD}^{(i)}(\gamma_{SR}^{(i)} + 1)^2 + \gamma_{SR}^{(i)}\gamma_{RD}^{(k)}(\gamma_{SR}^{(i)} + 1 + \gamma_{RD}^{(k)})} \right) \quad (11)$$

so that

$$\mathbf{y}_D = \mathbf{h}s + \mathbf{n}. \quad (10)$$

We assume the destination \mathcal{D} now applies a linear filter \mathbf{w} to \mathbf{y}_D to obtain an estimate of s . Although suboptimal, we will see later that in some cases the destination may wish to apply MRC ($\mathbf{w} = \mathbf{h}$) on \mathbf{y}_D , and doing so would result in the post-processing signal-to-noise ratio $\gamma^{(i)}$ of (11) at the top of the page. In this form, it is easy to see that, if

$$\left\{ \gamma_{SR}^{(i)} < \gamma_{SD}^{(i)} \right\} \cap \left\{ \gamma_{RD}^{(k)} > \gamma_{SD}^{(i)}(\gamma_{SR}^{(i)} + 1)/(\gamma_{SD}^{(i)} - \gamma_{SR}^{(i)}) \right\}, \quad (12)$$

then $\gamma^{(i)} < \gamma_{SD}^{(i)}$ and relaying is worsening performance. This occurs when the SNR from \mathcal{R} to \mathcal{D} is very good relative to the others, and the SNR from \mathcal{S} to \mathcal{R} is worse than the direct SNR. Effectively, the \mathcal{R} to \mathcal{D} channel is dominating the received signal, but it consists of mostly noise relative to the direct signal. Recall that MRC is only optimal when the observations contain the same noise variance [9]. Because of the amplified noise at \mathcal{R} , this is not the case here. In this case, one can show that the optimal receive filter in the minimum mean-squared error (MMSE) sense is

$$\mathbf{w} = \mathbf{R}_{\mathbf{y}_D}^{-1} \mathbf{R}_{\mathbf{y}_D s}, \quad (13)$$

where $\mathbf{R}_{\mathbf{y}_D} = \mathbb{E}\{\mathbf{y}_D \mathbf{y}_D^*\}$ and $\mathbf{R}_{\mathbf{y}_D s} = \mathbb{E}\{\mathbf{y}_D s^*\}$. The post-processing SNR is then

$$\gamma^{(i)} = \gamma_{SD}^{(i)} + \frac{\gamma_{SR}^{(i)}\gamma_{RD}^{(k)}}{\gamma_{SR}^{(i)} + \gamma_{RD}^{(k)} + 1}. \quad (14)$$

Note that this requires the destination to have knowledge of $\|\mathbf{h}_{SR}\|$. If this is not possible, suboptimal MRC resulting in the SNR of (11) may be used instead, which requires less training. A method for obtaining this CSI is presented in Section III.

Note that in (14), for fixed $\gamma_{SD}^{(i)}$ and $\gamma_{SR}^{(i)}$, $\gamma^{(i)}$ is maximized when $\gamma_{RD}^{(k)}$ is maximized. Thus, the antenna selection at the relay is independent of the selection at the source, and we can substitute the index of the optimal relay transmit antenna k_o in for k in all subsequent equations. The same cannot be said of the regular MRC equation (11).

Finally, we note that antenna selection at the relay is suboptimal, and the optimal strategy in this case is intuitive; since the SNR expression (14) is the addition of the independent SNR terms for the parallel

channels to the destination from the source, the relay should apply a filter that maximizes the SNR to the destination. One can show that this filter is $\mathbf{W} = \mathbf{v}^{(1)}(\mathbf{h}_{SR}^{(i_o)})^*$, where $\mathbf{v}^{(1)}$ is the right singular vector of \mathbf{H}_{RD} corresponding to its largest singular value, and i_o is the index of the source antenna that maximizes (14). Intuitively, \mathbf{W} is the combination of a receive filter matched to \mathbf{H}_{SR} and a transmit beamforming vector matched to \mathbf{H}_{RD} . Implementing this filter would require perfect knowledge of \mathbf{H}_{RD} at the relay and an SVD operation. All of our results hold with this optimal strategy, with $\|\mathbf{h}_{RD}^{(k_o)}\|^2$ replaced with $\lambda_{RD} = \sigma_{RD}^2$, the square of the largest singular value of \mathbf{H}_{RD} .

III. TRAINING AND LIMITED FEEDBACK

We now discuss how channel state information might be obtained in the channel of interest so that a reliable antenna selection strategy may be implemented. All three channels need to be estimated at their respective receivers; this can be accomplished using previously studied MIMO training methods. Only knowledge of the link SNRs (i.e., $\gamma_{XY}^{(i)}$'s) is required for transmit antenna selection. Therefore a low complexity signal, such as a short narrowband tone, may be used for estimating SNR to choose an antenna to train from. This is first sent from \mathcal{R} to \mathcal{D} from each relay antenna. \mathcal{D} then feeds back which antenna \mathcal{R} should use to transmit, and, from this antenna, a training sequence suitable for channel estimation is sent to the destination. The source repeats this process with its transmit antennas, with the relay forwarding its received signal on its optimal antenna. This way, the destination can estimate the SNR between the source and relay to perform MMSE combination as described earlier.

The destination finds (14) for each source antenna, then feeds back to the source the index of the antenna that resulted in the largest $\gamma^{(i)}$. The source then transmits a training sequence from this antenna, which does not need to be forwarded by the relay. This process requires $\log(N_S N_D)$ bits of feedback, two time slots of training, and $N_R + 2N_S$ time slots for SNR estimation. Minimizing the time required for SNR estimation is thus important for this feedback strategy.

IV. DIVERSITY ANALYSIS

Antenna selection is used to exploit the diversity gain available in the channel. Using (14) we now show that this strategy achieves full diversity gain. We first give an upper bound on the diversity order of the half-duplex MIMO relay channel when the source and destination transmit orthogonally in equal time slots. Yuksel and Erkip [6] have derived this result for arbitrary time sharing when the source is allowed to transmit in the second time slot, so this result is a special case of their derivation. This derivation is included here to prove that our added restrictions (i.e., equal transmission times, source silent in the

second time slot) do not decrease the maximum diversity order of the channel. We first define

$$I_{BC} = I(S; Y_R, Y_{D,1}) \quad (15)$$

$$I_1 = I(S; Y_{D,1}) \quad (16)$$

$$I_2 = I(S_R; Y_{D,2}) \quad (17)$$

$$I_{MAC} = I_1 + I_2, \quad (18)$$

where S is the random variable corresponding to the transmitted signal from the source, Y_R is the received signal at the relay, $Y_{D,n}$ is the received signal at the destination in the n th time slot, and S_R is the transmitted signal at the relay. Using equations (27) and (28) in [6] with $t = 0.5$ and the source not transmitting in the second time slot,

$$I(S; Y_D) \leq 0.5 \min\{I_{BC}, I_{MAC}\}. \quad (19)$$

Now we can bound the probability of outage for a fixed I_0 as

$$\begin{aligned} P_{out} &= \Pr\{I(S; Y_D) < I_0\} \\ &\geq \Pr\{0.5 \min\{I_{BC}, I_{MAC}\} < I_0\}. \end{aligned} \quad (20)$$

The event where the minimum of two variables is less than a constant is equivalent to the union of the events that each of the variables is less than the constant. Defining $P_{out,BC} = \Pr(I_{BC} < 2I_0)$, and similarly for $P_{out,MAC}$, we can write

$$P_{out} \geq \Pr\left(\{I_{BC} < 2I_0\} \cup \{I_{MAC} < 2I_0\}\right) \quad (21)$$

$$\begin{aligned} &= P_{out,BC} + P_{out,MAC} - \\ &\quad \Pr\left(\{I_{BC} < 2I_0\} \cap \{I_{MAC} < 2I_0\}\right). \end{aligned} \quad (22)$$

Recall from (18) that I_{MAC} is the sum of two nonnegative random variables. Such a sum is always less than or equal to twice the maximum of the two random variables. Then, by making the codebook for S_R independent from that of S , and defining $P_{out,I_1} = \Pr(I_1 < I_0)$ and P_{out,I_2} similarly,

$$\begin{aligned} P_{out} &\geq P_{out,BC} + \Pr\{\max\{I_1, I_2\} < I_0\} - \\ &\quad \Pr\left(\{I_{BC} < 2I_0\} \cap \{I_{MAC} < 2I_0\}\right) \end{aligned} \quad (23)$$

$$\begin{aligned} &\geq P_{out,BC} + P_{out,I_1}P_{out,I_2} - \\ &\quad \Pr\left(\{I_{BC} < 2I_0\} \cap \{I_{MAC} < 2I_0\}\right). \end{aligned} \quad (24)$$

Conversely, the sum of I_1 and I_2 is always greater than the maximum of the two. Also, note from (15) and (16) that $I_{BC} \geq I_1$ so that

$$\begin{aligned}
 P_{out} &\geq P_{out,BC} + P_{out,I_1}P_{out,I_2} - \\
 &\quad \Pr\left(\{I_{BC} < 2I_0\} \cap \{\max\{I_1, I_2\} < 2I_0\}\right) \\
 &= P_{out,BC} + P_{out,I_1}P_{out,I_2} - \\
 &\quad \Pr\{\max\{I_{BC}, I_2\} < 2I_0\}.
 \end{aligned} \tag{25}$$

Finally, again assuming independent channels on all links,

$$\begin{aligned}
 P_{out} &\geq P_{out,BC} + P_{out,I_1}P_{out,I_2} - \\
 &\quad P_{out,BC}\Pr\{I_2 < 2I_0\}.
 \end{aligned} \tag{26}$$

From MIMO information theory we know that (see [6], Sec. III and IV)

$$\Pr\{I_{BC} < c\} \doteq N_S(N_R + N_D) \tag{27}$$

$$\Pr\{I_1 < c\} \doteq N_S N_D \tag{28}$$

$$\Pr\{I_2 < c\} \doteq N_R N_D, \tag{29}$$

for all $c \in \mathbb{R}$. Thus, the last term in (26) will decay as $N_S N_R + N_S N_D + N_R N_D$ with log SNR and is thus irrelevant to the diversity analysis. The first term will decay as $N_S(N_R + N_D)$, while the second term decays as $N_D(N_S + N_R)$, so that

$$P_{out} \stackrel{\cdot}{\leq} N_S N_D + N_R \min\{N_S, N_D\}. \tag{30}$$

We now derive a lower bound on the diversity order of optimal antenna selection in flat i.i.d. Rayleigh fading by using (14). First we define

$$\gamma_{SRD}^{(i,k_o)} = \gamma_{SR}^{(i)} \gamma_{RD}^{(k_o)} / (\gamma_{SR}^{(i)} + \gamma_{RD}^{(k_o)} + 1). \tag{31}$$

Since we choose the source transmit antenna that maximizes the SNR γ at the destination,

$$\begin{aligned}
 P_{out} &= \Pr\{\gamma < \gamma_0\} \\
 &= \Pr\{\max_i \{\gamma^{(i)}\} < \gamma_0\} \\
 &= \Pr\{\max_i \{\gamma_{SD}^{(i)} + \gamma_{SRD}^{(i,k_o)}\} < \gamma_0\}.
 \end{aligned} \tag{32}$$

As before, the sum of two random variables is greater than the maximum of the two.

$$\begin{aligned} P_{out} &\leq \Pr\{\max_i\{\max\{\gamma_{SD}^{(i)}, \gamma_{SRD}^{(i,k_o)}\}\} < \gamma_0\} \\ &= \Pr\{\max_i\{\gamma_{SD}^{(i)}, \gamma_{SRD}^{(i,k_o)}\} < \gamma_0\}. \end{aligned} \quad (33)$$

Since each channel is mutually independent of the others, and the channel from each source antenna to the destination is also independent from the others, we define $P_{out,SD} = \Pr(\gamma_{SD}^{(1)} < \gamma_0)$, thus

$$\begin{aligned} P_{out} &\leq \Pr\{\max_i\{\gamma_{SD}^{(i)}\} < \gamma_0\} \Pr\{\max_i\{\gamma_{SRD}^{(i,k_o)}\} < \gamma_0\} \\ &= (P_{out,SD})^{N_S} \Pr\{\max_i\{\gamma_{SRD}^{(i,k_o)}\} < \gamma_0\}. \end{aligned} \quad (34)$$

Now define

$$\gamma_{M,i} = \min\{\gamma_{SR}^{(i)}, \gamma_{RD}^{(k_o)}\}. \quad (35)$$

If $\gamma_{M,i} \geq 1$, then $\gamma_{SRD}^{(i,k_o)} > \gamma_{M,i}/3$. Otherwise, $\gamma_{SRD}^{(i,k_o)} > (\gamma_{M,i})^2/3$. In either case, since γ_0 is arbitrary, we let $\gamma_0 > 1/3$ and proceed¹

$$P_{out} < (P_{out,SD})^{N_S} \Pr\{\max_i\{\gamma_{M,i}\} < 3\gamma_0\}. \quad (36)$$

We can again split up the minimum event into a union:

$$\begin{aligned} P_{out} &< (P_{out,SD})^{N_S} \times \\ &\Pr(\{\max_i\{\gamma_{SR}^{(i)}\} < 3\gamma_0\} \cup \{\max_i\{\gamma_{RD}^{(k_o)}\} < 3\gamma_0\}) \\ &= (P_{out,SD})^{N_S} \times \\ &\left[\Pr\{\max_i\{\gamma_{SR}^{(i)}\} < 3\gamma_0\} + \Pr\{\gamma_{RD}^{(k_o)} < 3\gamma_0\} - \right. \\ &\left. \Pr\{\max_i\{\gamma_{SR}^{(i)}\} < 3\gamma_0\} \Pr\{\gamma_{RD}^{(k_o)} < 3\gamma_0\} \right]. \end{aligned} \quad (37)$$

Again, since the channels between each source transmit antenna and the relay are independent, we define

$P_{out,SR} = \Pr(\gamma_{SR}^{(1)} < 3\gamma_0)$ and $P_{out,RD} = \Pr(\gamma_{RD}^{(k_o)} < 3\gamma_0)$, and

$$\begin{aligned} P_{out} &< (P_{out,SD})^{N_S} \times \\ &\left[(P_{out,SR})^{N_S} + P_{out,RD} - \right. \\ &\left. (P_{out,SR})^{N_S} P_{out,RD} \right], \end{aligned} \quad (38)$$

¹Since P_{out} is monotone increasing with increasing γ_0 , no loss in generality occurs by assuming $\gamma_0 > 1/3$. For example, let $\gamma_L < 1/3$. Then $\Pr(\gamma < \gamma_L) < \Pr(\gamma < \gamma_0)$. Thus, if $P_{out} \doteq d$, then $\Pr(\gamma < \gamma_L) \dot{\geq} d$.

where again the last term will decay much quicker than the others and can be ignored. The first term, after multiplication, will decay as $N_S N_D + N_S N_R$, while the second term decays as $N_S N_D + N_R N_D$. Thus,

$$P_{out} \stackrel{\cdot}{\geq} N_S N_D + N_R \min\{N_S, N_D\}. \quad (39)$$

Combining (39) and (30) we see that the proposed antenna selection achieves the full diversity gain in the channel.

V. SIMULATION RESULTS

We present a simple simulation to compare to a recent result on limited feedback beamforming [8]. For each case shown, we simulate the relay channel with $N_S = N_R = N_D = 3$ using BPSK modulation and an i.i.d. Rayleigh channel at each link. Bit error rate (BER) is the metric of interest. Figure 2 gives the results for $\mathbb{E}\{\gamma_{SR}^{(i)}\} = \mathbb{E}\{\gamma_{RD}^{(k)}\} = 2$ dB for various $\mathbb{E}\{\gamma_{SD}^{(i)}\}$. Note that this graph corresponds exactly to Fig. 9 in [8], and we have included their results for a Grassmannian codebook with more than 20 bits of feedback. Using antenna selection at both \mathcal{S} and \mathcal{R} requires 4 bits in this case and results in a loss of approximately 1 dB at high SNR.

The theoretical lower bound of Figure 2 is when the source can simultaneously beamform the BPSK symbols to both the relay and destination; obviously this is an impossible task. The “optimal” performance curve was found numerically in [8] using gradient descent to find a local optimum.

Figure 3 shows the BER of uncoded BPSK versus E_S/N_0 for a relay channel with two antennas at each node. Note that increasing E_S/N_0 implies an increase in SNR at *each link* (recall that noise terms are normalized and $\mathbb{E}|s|^2 = E_S = \text{SNR}$). The figure was generated using Monte Carlo simulations using 10^8 channel realizations for accuracy at high SNR, and demonstrates that antenna selection achieves the maximum diversity order available in the channel.

VI. CONCLUSION

We explored antenna selection as a practical way of achieving the full diversity order of the nonregenerative MIMO relay channel. It was shown to achieve this diversity with a small SNR penalty relative to Grassmannian codebooks.

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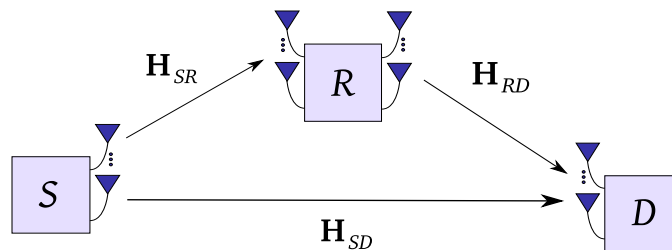


Fig. 1. The system model used in this letter. The source transmits in the first time slot, and the relay transmits in the second time slot. The relay is shown with separate transmit and receive antennas for convenience; this assumption is not made in the analysis.

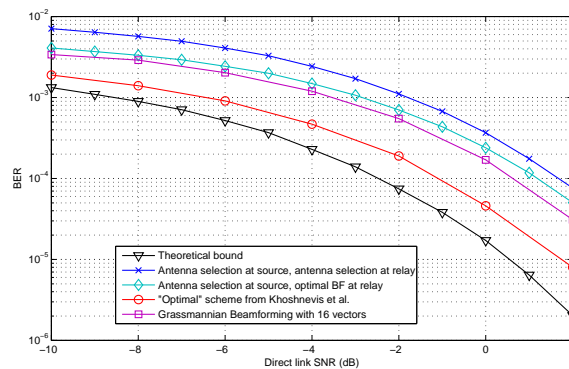


Fig. 2. BER performance for several MIMO amplify-and-forward beamforming strategies.

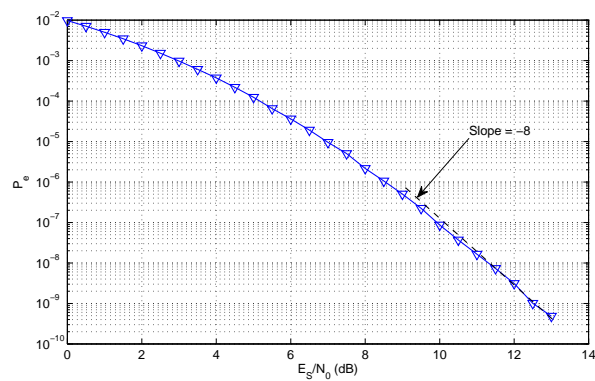


Fig. 3. BER performance for uncoded BPSK versus E_S/N_0 for i.i.d. Rayleigh fading. At high SNR, the slope of the curve approaches $-N_S(N_R + N_D) = -N_D(N_S + N_R) = -8$, which, as shown in Section IV, is the full diversity order of the channel.